GLOBAL INSPECTION GAMES
COMMON SHOCKS, ASYMMETRIC INFORMATION
AND AN APPLICATION TO TAX EVASION

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Abstract

Consider an "inspection game" in which resources are allocated based on the messages sent by potential beneficiaries and verification of messages is costly. If senders' types are correlated (e.g., if they are subject to a common shock), the inspector’s optimal strategy consists in inspecting senders with a probability that depends not only on their own message, but also on all other senders’ messages. Such policy generates a coordination game among senders, who then face both strategic uncertainty -about the equilibrium that will be selected- and fundamental uncertainty -about the type of inspector they face. Thus the situation can be realistically modelled as a global game that yields a unique and usually interior equilibrium which is consistent with empirical evidence.

The model can be applied to several areas of economic interest such as tax evasion, the regulation of industries and the allocation of welfare benefits, among others.

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1 Introduction

All over the world, transfers are made based on the reports made by potential beneficiaries: taxpayers file tax returns, welfare recipients declare themselves “needy” (poor, unemployed, ill, etc.), firms in regulated industries provide the regulator with information about their cost structures, etc. However, verifying self-reports is often costly for the principal: carrying out a tax audit, authenticating the status of a welfare claimant and finding out the cost structure of a firm require time, effort and resources, all of which have significant opportunity costs.

The problem behind these examples is, at the end of the day, a simple inspection problem. The tax/welfare/regulatory agency only has to ask itself the question: “which taxpayers/claimants/firms should be audited and which ones should not?” This is, as well, the question we will address in this paper. We will use the tax agency’s problem as a leading example throughout the paper, but the analysis can be easily adapted to the problem of the welfare, regulatory or any other generic inspection agency.

The tax evasion literature suggests a simple answer to the problem: the “cut-off” auditing policy (Reinganum and Wilde (1985)). It relies on the fact that tax agencies worldwide use observable characteristics of taxpayers to partition the population into fairly homogeneous categories in order to better estimate their incomes: \(\text{ceteris paribus}\), those who declare well below the estimate are likely to be evaders and are audited, while those who declare about or above it are likely to be compliant taxpayers and are not inspected.

The “cut-off” auditing policy, however, can lead to systematic mistargeting in the presence of common shocks: in good years the category would be under-audited (e.g., bars and pubs in a heat-wave); in bad years it would be over-audited (e.g., chicken-breeders in an avian-flu outbreak). To avoid this problem, the agency needs contemporaneous data correlated with the common shock. We examine the possibility of using the profile of declarations of the taxpayers in a category as a signal of the shock experienced by them and show that, when the agency faces a taxpayer who declares low income, the optimal auditing strategy is (weakly) increasing in other taxpayers’ declarations. Intuitively, the higher these declarations, the more likely the shock is a positive one, and so the more likely that someone who declares low income is an evader. Precisely this type of reasoning is presumed to be behind the method used by the Internal Revenue Service’s “Discriminant Index Function” (DIF) to determine which taxpayers to audit.\(^1\) This policy introduces a negative externality among taxpayers, one that would not exist otherwise: if someone increased her declaration, everyone else’s probability of detection would increase. This changes the nature of the evasion problem by creating a coordination game among agents: each one of them has incentives to evade if most people evade as well, and prefers to comply if most of the rest are compliant.

\(^1\) On page 301, Alm and McKee (2004) say: “(...) a taxpayer’s probability of audit is based not only upon his or her reporting choices, but also upon these choices relative to other taxpayers in the cohort. In short, there is a taxpayer-taxpayer game that determines each individual’s chances of audit selection.”
We avoid the coordination game’s multiplicity of equilibria and associated policy design problems by modeling the situation as a **global game**. Specifically, we model the agency’s innate “toughness” with respect to evasion as a parameter that is its private information, enters its objective function and affects its optimal policy: *ceteris paribus*, tougher agencies will audit more intensively than softer ones. Thus, taxpayers need to estimate it in order to decide how much income to declare and they do it based on the information available to them: each taxpayer’s previous experiences, conversations with friends and colleagues, and interpretation of media news constitute noisy signals of the tax agency’s type and are taxpayers’ private information. As signals are heterogeneous, different taxpayers perceive their situations as different from each other’s, and yet every one of them follows the same income declaration strategy. This leads to the survival of only one equilibrium in which (usually) some people evade and others comply, a result that is empirically supported and yet unlikely to be predicted by other tax evasion models.

Previous research on the tax evasion area (started by Allingham and Sandmo (1972) and surveyed by Cowell (1990) and Andreoni et al. (1998)) did analyse the effect of asymmetric information on the tax evasion game. Some focused on the uncertainty that taxpayers face about which equilibrium of the coordination game will be selected (“strategic uncertainty”), usually generated by psychological and/or social externalities (Benjamini and Maital (1985), Fortin et al. (2007), etc.). Others centered on the “fundamental uncertainty” faced by taxpayers with respect to the type of agency (Scotchmer and Slemrod (1989), Stella (1991), etc.). Unlike them, the present study considers both types of uncertainty and thus models the situation as a global game (Carlsson and van Damme (1993), Morris and Shin (2002b)).

The closest references to the present article are Alm and McKee (2004), Basseto and Phelan (2008) and Kim (2005). The first one is a laboratory experiment where the *(ad hoc)* auditing policy is contingent on the distribution of income declarations, while the second and third ones use the global game technique to determine the optimal tax system and the auditing policy, respectively. This paper presents a theoretical analysis in which—unlike the laboratory experiment—the agency’s optimal strategy is derived instead of assumed. The other two studies also employ the global game technique that we use here, but while Basseto and Phelan (2008) are concerned with the optimal tax system as designed by a government, this article focuses only on the targeting aspect of *one* of the agencies of the government. Finally, Kim (2005) generates the strategic interaction among taxpayers by adding a psychological cost (“stigma”) to their utility functions, whereas in my case it is the result of a rational tax agency that sets its auditing policy to maximize its objective function.

Regarding topics other than tax evasion, the informational effect of using other players’ information is well known, from relative performance compensation schemes (Lazear (1989) and Gibbons and Murphy (1990) among others) to yardstick competition mechanisms (Shleifer (1985), etc.). And the global games technique is becoming a popular methodology: Gilpatric et al. (2011) and Karp (2008), for example, apply it to environmental regulation, while Manz
(2009) deals with deposit insurance coverage. But, to the best of our knowledge, nobody found that the informational effect creates a coordination game that, in turn, is solved with the global games technique.

The rest of the paper is organized as follows: the model is presented in section 2. Results are shown in section 3 and discussed in section 4. Section 5 concludes.

2 Model

The model focuses on the interaction between a tax agency and the taxpayers within a given category. For simplicity, we will use “population of taxpayers” and “common shocks” to indicate the members of the category and the shocks faced by them, and not those of the whole population (i.e., the set which is the union of all categories), unless indicated otherwise.

2.1 Actors and strategies

There is a continuum of taxpayers, uniformly distributed on the $I := [0, 1]$ segment and indexed by $i \in I$. Each taxpayer’s strategy consists of her income declaration $d_i$. There is one tax agency. Its strategy consists of a vector of probabilities of detection, one for each taxpayer $a_i \forall i \in I$.

2.2 Timing

The situation is modelled as a one-shot game and its timing is as follows:

1. **Type Allocation Stage**: Nature determines the type of each actor. Types are private information: the agency learns its type $\lambda$ (its “toughness”) and each taxpayer learns her income $y_i$ and signal $s_i$.

2. **Declaration Stage**: Each taxpayer submits her income declaration $d_i$ and pays taxes accordingly.

3. **Inspection Stage**: The agency observes the vector of declarations $d$ and undertakes audits and collects fines (if any).
2.3 Informational setup

2.3.1 Common information set

At the beginning of the game, all actors share a “common information set” \( I^c \) which consists of the exogenous parameters of the problem (like the tax rate \( t \)), the probability distributions of private information variables, and the functional form of all players’ objective functions.

Objective functions

**Taxpayers**  Taxpayers are assumed risk-neutral, so their utility is:

\[
u(d_i, 1_i, y_i) = y_i - td_i - 1_i \phi_i \quad \forall i \in [0, 1]\] (1)

where \( y_i \in \{0, 1\} \) is taxpayer \( i \)'s gross (taxable) income, \( t \in (0, 1) \) is the income tax rate, \( d_i \in \{0, 1\} \) is agent \( i \)'s income declaration, \( 1_i \in \{0, 1\} \) is an indicator function that takes the value 1 if taxpayer \( i \) is audited and 0 if she is not, and \( \phi_i \) is the fine agent \( i \) should pay if audited and found guilty, defined as

\[
\phi_i := \Phi(d_i, y_i) = (1 + \zeta) t (1 - d_i) y_i
\] (2)

where \( \zeta \in (0, 1) \) is the surcharge rate that has to be paid by a caught evader on every dollar of evaded taxes.\(^3\)\(^4\) If \( a_i \) is the probability that taxpayer \( i \) is audited, then her expected utility function simplifies to

\[
u(d_i, a_i, y_i) = y_i - td_i - a_i (1 + \zeta) t (1 - d_i) y_i \quad \forall i \in I
\] (4)

**Tax Agency**  Narrowly defined, a tax agency’s objective is to raise revenue. More generally, its problem consists in determining which taxpayers should be audited and which ones should not. In other words, its goal is to minimize targeting errors.

These errors can be of two types: **Zeal** errors and **Negligence** errors. A zeal error takes place when a compliant taxpayer is audited. A negligence error occurs when an evader is

\(^2\)The assumption of a proportional tax system is not crucial. Results can be extended to cases in which the tax system is progressive.

\(^3\)The IRS applies rates between 20% (misconduct) and 75% (fraud) (Andreoni et al. (1998)), so \( \zeta \in (0, 1) \) covers the relevant range. It is assumed that \( (1 + \zeta) t < 1 \), such that the fine if caught evading does not exhaust a high-income person’s income.

\(^4\)The generic formula of the fine \( \phi_i \) is given by

\[
\phi_i := \Phi(d_i, y_i) = \begin{cases} f(y_i - d_i) & \text{if } d_i < y_i \\ 0 & \text{otherwise} \end{cases}
\] (3)

where \( f := (1 + \zeta) t \) is the fine per dollar of evaded taxes. Since both incomes and declarations can take only two values (0 or 1), the fine function simplifies to the one in equation 2.
not caught.\(^5\) Formally,
\[
N(d_i,1_i, y_i) := (1-1_i)(1-d_i)y_i \tag{5}
\]
\[
Z(d_i,1_i, y_i) := 1_i[1-(1-d_i)y_i] \tag{6}
\]
where a negligence error occurs only when an evader (someone with high income \(y_i = 1\) who declares low income \(d_i = 0\)) is not audited \((1_i = 0)\); and a zeal error occurs only when a compliant taxpayer (when \(d_i = y_i \in \{0,1\}\)) is audited \((1_i = 1)\).\(^6\) The expected negligence and zeal errors are therefore:
\[
N_i := a_i N(d_i,1_i, y_i) + (1-a_i) N(d_i,0, y_i) = (1-a_i)(1-d_i)y_i \tag{9}
\]
\[
Z_i := a_i Z(d_i,1_i, y_i) + (1-a_i) Z(d_i,0, y_i) = a_i[1-(1-d_i)y_i] \tag{10}
\]
respectively.

Clearly, different agencies can value each kind of error differently. If \(\mu \in [0,1]\) is defined as the weight attached by the agency to negligence errors, the expected loss inflicted by agent \(i\) on an agency of type \(\lambda\) can be expressed as
\[
L_i := L(a_i,d_i, y_i) := \mu N_i + (1-\mu) Z_i \tag{11}
\]
Aggregating over all taxpayers, we obtain
\[
L(a,d,y) := \int_{1 \in \lambda := [0,1]} [\mu N_i + (1-\mu) Z_i] \, di \tag{12}
\]
The tax agency faces a budget constraint that limits the number of audits it can undertake to the ones allowed by the agency’s budget \(b(\lambda)\).\(^7\) Regarding the latter, we assume that

\(^5\) The definition of a negligence error requires the following assumption:

**Assumption 1** An audit’s cost is lower than the fine paid by an evader. Formally, \(c < (1 + \gamma) t\).

Intuitively, if the cost of the audit were greater than the fine, then the agency is better off not auditing the evader, and so not auditing her would not be a targeting mistake. This case, however, is uninteresting because the problem becomes trivial: the tax agency would never audit anyone because the fine would never be high enough as to compensate for the cost of the audit.

\(^6\) The generic formula for negligence and zeal errors are given by
\[
N_i := \begin{cases} 
1 & \text{if } p_i = 1 \text{ and } 1_i = 0 \\
0 & \text{otherwise}
\end{cases} \quad Z_i := \begin{cases} 
1 & \text{if } p_i = 0 \text{ and } 1_i = 1 \\
0 & \text{otherwise}
\end{cases} \tag{7}
\]
where \(p_i\) is the “profitability” of auditing taxpayer \(i\) and is defined as
\[
p_i := \begin{cases} 
1 & \text{if } y_i = 1 \text{ and } d_i = 0 \\
0 & \text{otherwise}
\end{cases} \tag{8}
\]
That is, an audit is profitable \((p_i = 1)\) only when it targets an evader (thanks to assumption 1). Hence, a negligence error \((N_i)\) occurs when a profitable audit \((p_i = 1)\) is not undertaken \((1_i = 0)\); and a zeal error \((Z_i)\) occurs when a non-profitable audit \((p_i = 0)\) is undertaken \((1_i = 1)\).

\(^7\) More precisely, \(b(\lambda)\) is the “effective budget” of the tax agency, meaning it is the maximum number of audits (as a fraction of the total population) that the tax agency can undertake in the period given its
The average declaration $D^*$ is given by the following expression:

**Assumption 2** *The tax agency’s budget is given by $b(\lambda) = \max\{\lambda; 0\}***

This assumption formalizes the idea that tougher agencies can audit more intensively than softer ones.

**Probability distributions of private information variables**

**Incomes** are assumed perfectly correlated to reflect the fact that **common shocks** affect similar agents in similar ways:

$$y_i = y \quad \forall i \in I$$  \hspace{1cm} (13)

“Good years” ($y = 1$) occur with probability $\gamma \in (0, 1)$ and “bad years” with probability $1 - \gamma$.\(^8\) Formally,

$$\Pr(y = 1) = \gamma$$  \hspace{1cm} (14)

The **agency’s type** $\lambda \in \mathbb{R}$ is a non-manipulable characteristic of the agency that affects the tax agency’s auditing policy. It is a continuous variable that is distributed according to a continuous and strictly increasing cumulative distribution function (CDF) $J(\lambda)$ and is independent of the income shock. Economically, it indicates how “tough” the agency is: *ceteris paribus*, tougher agencies (those with higher $\lambda$s) audit more frequently than softer ones.

**Taxpayers’ signals** $s_i$ convey information about the tax agency’s type $\lambda$ and are, on average, correct. They reflect the information about the agency’s type that taxpayers get from all available sources: media news, previous experiences, conversations with colleagues and friends, etc. Formally,

$$s_i := \lambda + \varepsilon_i \quad \forall i \in I$$  \hspace{1cm} (15)

where $\varepsilon_i$ is the error term, which is assumed to be white noise ($E(\varepsilon_i) = 0 \ \forall i$), with CDF $F(\varepsilon_i)$, uniformly distributed on the $[-\varepsilon, \varepsilon]$ segment, and independent of income $y_i$, other taxpayers’ errors $\varepsilon_{j \neq i}$ and the tax agency’s type $\lambda$.

**2.3.2 Private information sets**

**Taxpayers** At the end of stage 1 taxpayer $i$ knows the realization of her private information variables, namely, her income $y_i$ and her private signal $s_i$. Furthermore, since all resources and type $\lambda$. For simplicity of exposition, we will however refer to it as simply “the budget”.\(^8\) Qualitative results are robust to the introduction of more levels of income and/or small idiosyncratic income shocks.
taxpayers know that the income distribution is degenerate, they know that every taxpayer has the same income \( y \) (\( y = 0 \) if \( y_i = 0 \) and \( y = 1 \) if \( y_i = 1 \)). Notice also that, from equation 15 and Bayes law, taxpayers know that the type of the agency \( \lambda \) is uniformly distributed with support \([s_i - \varepsilon, s_i + \varepsilon]\). Formally, the conditional probability distribution of \( \lambda \) is given by:

\[
G (\lambda|s_i) : \lambda|s_i \sim U [s_i - \varepsilon, s_i + \varepsilon] \quad \forall i \in I
\]  

(16)

Thus, taxpayer \( i \)'s private information set \( I_i^p \) at the moment of deciding her income declaration is: \( I_i^p = \{ y_i, s_i \} \). Her information set is therefore \( I_i = I_i^p \cup I^c \).

**Tax agency** At the end of stage 2 the *tax agency* knows the realization of her private information variable, its type \( \lambda \) (and thus its budget \( b (\lambda) \)). Also, given the timing of the game, it observes the vector of taxpayers’ income declarations \( d \) which consists of the declarations of every taxpayer in the population, \( d_i \in \{0,1\} \), \( \forall i \in I \). Given the dichotomous nature of the declarations, the vector of income declarations can be summarized by a sufficient statistics, namely, the average declaration \( D \in [0,1] \), which will be used henceforth and is defined as:

\[
D := \int_{i \in I} d(s_i, y_i) \, di
\]  

(17)

Analogously to the taxpayers’ case, from equation 15 and Bayes law, the agency knows that signals are uniformly distributed with support \([\lambda - \varepsilon, \lambda + \varepsilon]\). Formally, the conditional probability distribution of \( s_i \) is given by:

\[
H (s_i|\lambda) : s_i|\lambda \sim U [\lambda - \varepsilon, \lambda + \varepsilon]
\]  

(18)

Thus, the agency’s private information set at the moment of making her auditing decisions is \( I_{TA}^p = \{ \lambda, D \} \). Its information set is therefore \( I_{TA} = I_{TA}^p \cup I^c \).

### 2.4 Equilibrium concept

A perfect Bayesian equilibrium (PBE) consists of beliefs and strategies satisfying two conditions:

\textbf{(i)} at information sets on the equilibrium path, beliefs are determined by Bayes rule and the players’ equilibrium strategies; and

\textbf{(ii)} given the players’ beliefs, their strategies are sequentially rational.

\footnote{By construction, \( D \) coincides with the proportion of taxpayers who declare high income (in good years).}
2.4.1 Candidate equilibrium

Based on a standard result in the global games literature (Morris and Shin (2002a), Myatt et al. (2002), among others), we posit that, in good years, all taxpayers use the same optimal strategy, whose main feature is that it is a step function of the signal received. Formally,

Candidate Equilibrium The candidate equilibrium satisfies the (above mentioned) two conditions required for a PBE to exist and, more specifically, the solution to the taxpayer problem is as follows: \( \forall i \in I, \)

\[
\begin{align*}
  d^* (y_i, s_i) = \begin{cases}
    0 & \text{if } y = 0 \\
    0 & \text{if } y_i = 1 \text{ and } s_i < \Sigma \\
    \in [0, 1] & \text{if } y_i = 1 \text{ and } s_i = \Sigma \\
    1 & \text{if } y_i = 1 \text{ and } s_i > \Sigma 
  \end{cases}
\end{align*}
\]

and

\[
\Sigma \in (-\xi, \xi)
\]

Intuitively, the step function (last three lines of equation 19) is a reasonable assumption because: 1 income declarations can take only two values (\( d_i^* \in \{0, 1\} \forall i \in I \)); and 2 higher signals imply higher values of \( \lambda \) (i.e., a tougher agency) and thus a higher probability of being audited, which in turn increases taxpayers’ incentives to comply\(^\text{10}\). The optimal declaration strategy can thus be visualized as in table 1.

| TABLE 1 ABOUT HERE |

3 Results

3.1 Preliminaries

It is worth noticing the following result:\(^\text{11}\)

Proposition 1 In bad years, declaring low income is a strictly dominant strategy for all taxpayers. Formally, \( d_i^* (0, s_i) = 0 \forall i \in I \land \forall s_i \in \mathbb{R} \).

\(^{10}\)Clearly, this second argument relies on the assumption that a tougher agency will audit with (weakly) higher probability than a softer one, something that will be shown to be the case when we solve the tax agency’s problem in section 3.2.

\(^{11}\)All proofs are in the appendix.
The intuition is straightforward: while under-declaring is punished if discovered (first line of equation 3), over-declaring is never rewarded (second line of the same equation), and hence taxpayers have no incentive to over-declare.

The following result is a direct consequence of equation 19:

**Result 1** The average declaration $D^*$ is given by the following expression:

$$D^* (\lambda, y) = 1 - H (\Sigma|\lambda, y) = \begin{cases} 0  & \text{if } \lambda \leq \Sigma - \varepsilon \\ \frac{\lambda + \varepsilon - \Sigma}{\Sigma} y & \text{if } \Sigma - \varepsilon < \lambda \leq \Sigma + \varepsilon \\ \gamma & \text{if } D^* = 0 \quad \text{and} \quad \lambda \leq \Sigma - \varepsilon \\ y & \text{if } \Sigma + \varepsilon < \lambda \end{cases}$$  \hspace{1cm} (21)$$

The result simply says that the average declaration is equal to zero in bad years (it is immediate from proposition 1), while in good years it is equal to the proportion of people who declare high income, which is equal to the proportion of people who get signals higher than the threshold $\Sigma$ (immediate from equation 19). The solid line in figure 1 illustrates the result.

**FIGURE 1 ABOUT HERE**

Result 1, in turn, is crucial to prove the following lemma:

**Lemma 1** The probability of a good year conditional on the average declaration $D^*$ and the type of the agency $\lambda$ is given by

$$\pi (D^*, \lambda) := \Pr (y = 1|D^*, \lambda) = \begin{cases} 1 & \text{if } D^* > 0 \\ \gamma & \text{if } D^* = 0 \quad \text{and} \quad \lambda \leq \Sigma - \varepsilon \\ 0 & \text{if } D^* = 0 \quad \text{and} \quad \Sigma - \varepsilon < \lambda \end{cases}$$  \hspace{1cm} (22)$$

Intuitively, observing $D^* > 0$ (first line) implies that some people declared high income, and this can only happen if income is truly high.

On the other hand, when $D^* = 0$ two cases are possible:

i. when $\lambda \leq \Sigma - \varepsilon$ (the agency is very soft; second line of equation 22), then observing the average declaration does not provide any extra information to the agency because average declaration is zero both in good and bad years (see equation 21). Thus, the best estimate available to the observer is the unconditional probability of a good year, namely, $\gamma$ (as defined in equation 14).

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12Further insights are offered in the proof of the result on page 25.
ii. when $\Sigma - \varepsilon < \lambda$ (the agency is not very soft; third line of equation 22), the agency knows that the average declaration would be strictly positive if income were high (see equation 21), and so observing $D^* = 0$ indicates that income is truly low.

The following definition, lemmas and result will be useful later on. Definition 1 implicitly defines $\Lambda$ as the level of “toughness” of an agency such that its budget, $b(\Lambda)$, is just enough to audit every evader, $1 - D^*(\Lambda, 1)$. Lemma 2 posits that when the agency is so soft that its budget is zero, it does not audit anyone. Lemma 3 states that the proportion of low declarers that can be audited, $\frac{b(\Lambda)}{1-D}$, increases with the “toughness” of the agency, $\lambda$. Finally, result 2 ranks some important concepts.

**Definition 1** Define $\Lambda$ implicitly as:

$$\frac{b(\Lambda)}{1 - D^*(\Lambda, 1)} := 1 \quad (23)$$

**Lemma 2** If $\lambda < \Sigma - \varepsilon$, then $\min \{b(\lambda) ; 1\} = 0$.

**Lemma 3** If $0 < D \leq 1$, then $\min \left\{ \frac{b(\lambda)}{1-B} ; 1 \right\} = \begin{cases} 0 & \text{if } \lambda \leq 0 \\ \frac{2\varepsilon\Lambda}{\Sigma + \varepsilon} & \text{if } 0 \leq \lambda \leq \Lambda \text{ where } \Lambda \text{ is defined as in definition 1 and takes the value} \\ 1 & \text{if } \Lambda \leq \lambda \end{cases}$

**Result 2** It is the case that

$$\Sigma - \varepsilon < 0 < \Lambda < \Sigma + \varepsilon \quad (25)$$

### 3.2 Tax agency problem

At stage 3 the tax agency chooses the probability of auditing low declarations, $a^0$, and the probability of auditing high declarations, $a^1$, in order to minimize the expected loss function$^{13}$

$$ELF = [1 - \pi(D^*, \lambda)] (1 - \mu)a^0 + \pi(D^*, \lambda) \left[D^*(1 - \mu)a^1 + (1 - D^*)\mu(1 - a^0)\right] \quad (26)$$

subject to the budget constraint.

The expected loss function can be interpreted as follows. The agency believes true income is low with probability $1 - \pi(D^*, \lambda)$ and so it makes $a^0$ zeal errors (as many audits as it

$^{13}$The derivation of the Expected Loss Function is on page 24 in the Appendix.
undertakes), each one generating a loss of $1 - \mu$ (first term). With probability $\pi(D^*, \lambda)$ the agency believes true income is high and so it makes $D^* a^1$ zeal errors (a fraction $D^*$ of taxpayers declare high income and a fraction $a^1$ of them are audited) and $(1 - D^*) (1 - a^0)$ negligence errors (a fraction $(1 - D^*)$ of taxpayers declare low income and a fraction $1 - a^0$ of them are not audited) (second term).

The solution to the problem is summarized in the following proposition:

**Proposition 2** An agency’s optimal auditing strategy is as follows:

1. if a taxpayer declares high income ($d^*_i = 1$), do not audit her;
2. if a taxpayer declares low income ($d^*_i = 0$) and
   
   (a) everyone else also declares low income ($D^* = 0$), do not audit her;
   
   (b) someone else declares high income ($0 < D^* \leq 1$) and
      
      i. the agency’s type is soft ($0 \leq \lambda \leq 0$), do not audit her;
      
      ii. the agency’s type is medium ($0 \leq \lambda \leq \Lambda = \frac{\Sigma_{i=0}^\infty}{1 + 2\epsilon}$), audit her with probability $\frac{2\lambda}{2\epsilon + \lambda}$;
      
      iii. the agency’s type is tough ($\Lambda \leq \lambda$), audit her with probability 1.

Formally, for every taxpayer $i \in [0, 1]$,

$$a^*_i (d^*_i, D^*, \lambda) = \begin{cases} 0 & \text{if } d^*_i = 1 \\ 0 & \text{if } d^*_i = 0; D^* = 0 \\ 0 & \text{if } d^*_i = 0; 0 < D^* \leq 1; \lambda \leq 0 \\ \frac{2\lambda}{2\epsilon + \lambda} & \text{if } d^*_i = 0; 0 < D^* \leq 1; 0 \leq \lambda \leq \Lambda \\ 1 & \text{if } d^*_i = 0; 0 < D^* \leq 1; \Lambda \leq \lambda \\ \end{cases} \quad (27)$$

Intuitively, the proposition says that an agency’s optimal auditing decision with respect to a given taxpayer $i$ depends on the taxpayer’s decision $d^*_i$, the declarations of all other taxpayers (summarized by the average declaration $D^*$) and the agency’s type $\lambda$. When at least one person declares high income (and so $D^* > 0$), the tax agency knows for sure – thanks to the perfect correlation assumption – that the shock was a positive one (it was a “good year”), and so the optimal strategy consists in: (1) not auditing anyone who declares high income (only high-income taxpayers ever declare high income, and so their declarations are truthful – first line of equation 27), and (2) auditing as many taxpayers who declared low income as possible because they are evaders (lines 3 to 5 of equation 27, corresponding to the cases in which the agency’s budget allows it to audit none, some and all evaders, respectively).

When everyone declares low income ($D^* = 0$), there are two cases to consider:
1. If the agency is very soft (\(\lambda \leq \Sigma - \varepsilon\)), the agency’s budget is zero (from assumption 2 and result 2) and so it never audits.

2. If the agency is not very soft (\(\Sigma - \varepsilon < \lambda\)), then at least some taxpayers declare high income in good years (their signals are greater than the threshold \(\Sigma\)). Thus, the agency knows that \(D^* = 0\) if \(y = 0\) and \(D^* > 0\) if \(y = 1\), and it never audits when \(D^* = 0\).

Thus, both cases yield the same outcome: the agency’s optimal strategy when everyone declares low income is to audit no one: \(a^0 = 0\) if \(D^* = 0\) (line 2).

The tax agency’s optimal auditing strategy can thus be visualized as in table 2.

**TABLE 2 ABOUT HERE**

Figure 2, meanwhile, plots the probability of auditing a taxpayer who declares low income \(a^0\) when the average declaration is positive.\(^{14}\)

**FIGURE 2 ABOUT HERE**

The main comparative statics results are summarized in the following proposition:

**Proposition 3** The probability of auditing taxpayer \(i\) is:

1. a weakly increasing function of the type of the agency \(\lambda\);
2. a weakly decreasing function of taxpayer \(i\)’s declaration; and
3. a weakly increasing function of taxpayer \(j\)’s declaration for all \(j \neq i\).

Formally,

\[
\begin{align*}
(1) & & a_i^* (d_i^*, D^*, \lambda') \geq a_i^* (d_i^*, D^*, \lambda) & \forall \lambda' > \lambda \\
(2) & & a_i^* (d_i^{**}, D^*, \lambda) \leq a_i^* (d_i^*, D^*, \lambda) & \forall d_i^{**} > d_i^* \\
(3) & & a_i^* (d_i^{**}, D^{**}, \lambda) \geq a_i^* (d_i^*, D^*, \lambda) & \forall D^{**} > D^*
\end{align*}
\]

The first two parts of the proposition are rather non-surprising: tougher agencies audit more intensively than softer ones and agencies audit individuals who declare high income with a lower probability than those who declare low income (as is standard in tax evasion models).

\(^{14}\)The dotted line is the proportion of taxpayers who declare low income, \(1 - D\). The dashed line is the agency’s budget, \(b(\lambda)\). The solid line is \(a^0\), the audit probability when \(D > 0\).
The novelty of the present study is in part 3, which states that a loss-minimizing agency would use the information conveyed by the vector of income declarations (in this case, by the average declaration) when deciding its optimal policy. Intuitively, other taxpayers’ declarations provide contemporaneous information about the likelihood of a given income shock and therefore improve the targeting proficiency of the agency: it can perfectly distinguish between truthful and untruthful declarations when the average declaration is different from zero.

3.3 Taxpayer problem

At stage 2 each taxpayer chooses her income declaration $d_i$ in order to maximize her expected utility, conditional on her information set $I_i$. In good years, taxpayer $i$’s expected utility function is:

$$E_i(u_i) := E(u_i|I_i) = 1 - td_i - \phi_i \cdot \alpha_i$$

where $\phi_i$ is the fine (as defined in equation 2) and $\alpha_i := E_i[a_i^*(d_i, d_{-i}^*, \lambda)]$ is the taxpayer’s subjective belief about the probability of an audit (a belief that is, at the end of the day, a function of the information available to taxpayer $i$, namely, her income $y_i$ and her signal $s_i$).

Notice that while evasion ($d_i = 0$) is a risky action that yields expected utility $E_i(u_i^{\text{evasion}}) = [1 - (1 + \zeta) t \cdot \alpha_i^*(0, d_{-i}^*, \lambda)]$; compliance is a safe action that generates utility $u_i^{\text{compliance}} = (1 - t)$ with certainty. Therefore, taxpayer $i$’s optimal decision in good years $d_i^*(1, \alpha_i)$ depends on the comparison between the two:

$$d_i^* = \begin{cases} 
0 & \text{if } y_i = 0 \\
0 & \text{if } y_i = 1 \text{ and } \alpha_i < P \\
[0, 1] & \text{if } y_i = 1 \text{ and } \alpha_i = P \\
1 & \text{if } y_i = 1 \text{ and } \alpha_i > P 
\end{cases}$$

where

$$P := \frac{1}{1 + \zeta} \in \left(\frac{1}{2}, 1\right)$$

is the minimum probability of detection that can eliminate evasion.\(^{16}\)

Intuitively, in good years taxpayers evade only if their subjective belief about the probability of being audited is not too high. This implies that an agent’s declaration is (weakly) increasing in her expectation over the probability of detection, a standard result in the tax evasion literature:

**Result 3** In good years, taxpayer $i$’s declaration is a weakly increasing function of her

---

\(^{15}\)The bad-year scenario is dealt with in proposition 1.

\(^{16}\)The domain of $P$ is obtained from the domain of $\zeta$, as defined on page 5.
subjective belief about the probability of being audited. Formally, if $\alpha'_i > \alpha_i$, then $d'_i(\alpha'_i) \geq d'_i(\alpha_i)$.

It is straightforward to see the similarity between equations 19 and 32. In particular, for the indifference condition (between evading and not) to be the same it is necessary that the subjective belief of the taxpayer that receives a signal $\alpha_i(\Sigma)$, be equal to $P$.

**Proposition 4** The threshold level $\Sigma$ is defined by the expression

$$\alpha_i(\Sigma) := P$$

and in equilibrium it takes the value

$$\Sigma = (2P - 1)\varepsilon$$

The following corollary is an immediate consequence of equation 35:

**Corollary 1** The equilibrium threshold level $\Sigma$ satisfies the condition demanded by the candidate equilibrium. Formally, $\Sigma = (2P - 1)\varepsilon$ is such that $\Sigma \in (-\varepsilon, \varepsilon)$.

Proposition 4, however, is not enough to prove that the solution to the taxpayer problem (equation 32) coincides with the one suggested by the candidate equilibrium (equation 19). It requires the following proposition:

**Proposition 5** In good years, if the private signal $s_i$ received by taxpayer is higher than/equal to/lower than the threshold $\Sigma$, then she believes her probability of being audited $\alpha_i$ is higher than/equal to/lower than the probability of detection that eliminates evasion $P$, and therefore she complies/is indifferent between evading and complying/evades. Formally, $\forall i \in I$,

$$\text{if } s_i \geq \Sigma \text{ then } \alpha_i \geq P$$

The intuition is straightforward: the higher the signal received ($s_i := \lambda + \varepsilon_i$ from equation 15), the higher is the taxpayer’s belief about the tax agency’s type $\lambda$ and, consequently, the higher her expected probability of detection $\alpha_i$. This decreases the (subjective) expected return of evasion and makes compliance more attractive, which leads the taxpayer to (weakly) increase her income declaration.

There is, however, a second channel through which a higher signal $s_i$ leads to a higher declaration $d_i^*$: a high signal $s_i$ means that other taxpayers are also likely to receive high signals and to declare high income. This increases the average declaration $D^*$ and, consequently,
the expected probability of detection $\alpha_i$, which in turn makes compliance relatively more attractive. Hence, the following proposition results:

**Proposition 6** Taxpayers’ declarations are (weakly) strategic complements. Formally, for every $j \neq i$,

$$d_i^* (d_j^*) \geq d_i^* (d_j^*) \quad \forall d_j^* > d_j^* \wedge \forall j \neq i$$  \hspace{1cm} (37)

Intuitively, this proposition says that taxpayers are better off complying/evading when others comply/evade: the more people comply/evade, the more/less likely that evasion is discovered (more/less likely that the taxpayer is audited because there are relatively few/many people who declare low income) and the incentives to evade are lower/higher.

More importantly, this proposition transforms the nature of the game because it creates a coordination game among the taxpayers on top of the cat-and-mouse game that each one of them plays against the agency and that is usually the only one considered by the literature. The strategic complementarity between taxpayers’ declarations, however, is not an exogenous characteristic of the game (as in Kim (2005)), but rather one that is created by the agency in its attempt to minimize its targeting errors. Indeed, it is the fact that the agency’s optimal auditing strategy is an increasing function of other taxpayers’ declarations (equation 30) that generates the strategic complementarity between taxpayers’ declarations (equation 37). That is, a rational agency, willing to minimize its targeting-related losses, designs its optimal auditing strategy by introducing some strategic uncertainty (i.e., by creating a coordination game between taxpayers) that improves its ability to distinguish compliant from non-compliant agents and thus decreases the frequency of targeting mistakes.

### 3.4 Uniqueness of equilibrium: Global games technique

A priori, the generation of a coordination game among taxpayers does not look as a good idea for the agency because this kind of games present multiple equilibria, which make policy design a complicated matter. Nevertheless, this difficulty is overcome thanks to the presence of a second source of uncertainty (called “fundamental uncertainty”) that allows for the tax evasion problem to be modelled as a “global game” (Carlsson and van Damme (1993), Morris and Shin (2002b)).\(^{17}\)

This equilibrium-selection technique eliminates all but one equilibria by introducing heterogeneity in taxpayers’ information sets in the form of noisy private signals that convey information about the agency’s private information parameter $\lambda$ (the source of the “fundamental uncertainty”). Thus, taxpayers do not observe the true coordination game, but

\(^{17}\)In other applications (bank runs, currency crises, etc), this technique was criticized because of not taking into account the coordinating power of markets and prices (Atkeson (2000)). This criticism is greatly mitigated in the case of tax evasion, since there is no “insurance market against an audit” to aggregate information about the government’s type (the “fundamental”, in global games jargon).
slightly different versions of it. This is the case since taxpayers with different signals would work out different estimates of the agency’s type \( \lambda \) and the average declaration \( D^* \), and so of their probabilities of detection. The optimal declaration strategy, however, is one and the same for every “type” of taxpayer (equations 19 and 35). The rationale for this result goes along the lines described in the paragraph immediately before proposition 6: my own signal gives me information about the possible signals that other taxpayers may have received and, more importantly, about the signals that they cannot have received, thus allowing me to discard some strategies that they cannot have followed. The application of this process iteratively by every taxpayer leads to the elimination of all strictly dominated strategies and leaves only one optimal strategy to be followed by every taxpayer (Morris and Shin (2001)), namely, the ones in equations 19 and 35. As a consequence, the equilibrium is unique.

3.5 Characterization of equilibrium

Combining the results from tables 1 and 2, we can fully characterize the equilibrium as in table 3 (where \( \Sigma = (2P - 1)\varepsilon \) and \( \Lambda = \frac{2\varepsilon P}{1+2\varepsilon P} \)).

This table summarizes the results obtained throughout the paper: in bad years (when \( y = 0 \)) taxpayers declare low income and are not audited. In good years (when \( y = 1 \)), several scenarios are possible: (1) if the agency is very soft (when \( \lambda < \Sigma - \varepsilon \), so that every taxpayer receives a signal lower than \( \Sigma \)) then all taxpayers evade \( (D = 0 \) in both good and bad years) and the agency audits no one; (2) if the agency is very tough (when \( \lambda > \Sigma + \varepsilon \), so that every taxpayer receives a signal greater than \( \Sigma \)) then all taxpayers comply \( (D = 1) \) and the agency audits no one; (3) if the agency is neither very soft nor very tough (when \( \Sigma - \varepsilon < \lambda < \Sigma + \varepsilon \), so that some taxpayers receive signals lower than \( \Sigma \) and others receive signals greater than \( \Sigma \), then the latter comply and the former evade, and the agency only audits those who declare low income with probability \( \frac{\Sigma}{2\varepsilon P - \lambda} \) or 1 (depending on whether the budget is low, intermediate or high, respectively).

Using table 3, the payoffs of the actors are computed in table 4: In bad years taxpayers declare low income and pay no taxes, hence their utility is zero. When the agency is very soft \( (\lambda < \Sigma - \varepsilon) \) the agency never audits because it has no budget; and taxpayers declare low income in both cases, so that in bad years their utility is \( u_i = 0 \) and the agency makes no mistake, and in good years taxpayers get utility 1 and the agency does not audit any of them so its expected loss is \( \gamma \mu \). In all other cases, the agency can infer the true income, and so in bad years the agency audits no one and makes no mistake. In good years, there are several possible cases. When the agency is very tough \( (\lambda > \Sigma + \varepsilon) \) all taxpayers receive signals

\[ \frac{2\lambda}{2\varepsilon P - \lambda} \] or 1 (depending on whether the budget is low, intermediate or high, respectively).

\[
\text{TABLE 3 ABOUT HERE}
\]

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\[ \frac{2\lambda}{2\varepsilon P - \lambda} \] or 1 (depending on whether the budget is low, intermediate or high, respectively).

\[
\text{TABLE 3 ABOUT HERE}
\]
above the threshold $\Sigma$, comply and get utility $1 - t$, while the agency audits no one, makes no error and suffers no loss. In the other (intermediate) cases ($\Sigma - \varepsilon < \lambda < \Sigma + \varepsilon$), some taxpayers receive high signals, comply and get utility $1 - t$, some others receive low signals, evade and get utility $1 - \frac{1}{p}a^0$ (with $a^0$ taking the values indicated in the corresponding cases in table 3), and the agency does not audit any high-declarer (and thus makes 0 zeal errors), audits as many low-declarers as possible (and makes $(1 - D)(1 - a^0)$ negligence errors), and the expected loss is $\mu(1 - D)(1 - a^0)$.

TABLE 4 ABOUT HERE

3.6 Information precision

In this section we explore the effect of more precise signals on the players' equilibrium strategies and payoffs.\textsuperscript{19}

**Result 4** In good years, when signals become more precise, the average declaration weakly decreases if the agency is soft ($\lambda < 0$) and weakly increases otherwise. Formally, if $y = 1$, then

\[
\begin{align*}
&\text{if } \lambda < 0, \text{ then } D^*(\lambda; \varepsilon') \geq D^*(\lambda; \varepsilon) \quad \forall \varepsilon' > \varepsilon \\
&\text{if } \lambda > 0, \text{ then } D^*(\lambda; \varepsilon') \leq D^*(\lambda; \varepsilon) \quad \forall \varepsilon' > \varepsilon
\end{align*}
\]

(38)

The result highlights the fact that the impact of more precise information on the level of evasion ($1 - D^*$) depends on the type of the agency. This is at odds with previous studies, which usually find that better information leads to more evasion, via the argument that it decreases the risk borne by taxpayers who, assumed risk averse, have therefore more incentives to engage in the riskier activity, namely, evasion.

Though compelling, this argument cannot be applied to the present case because here agents are assumed risk neutral. Yet, what matters is that the relationship between compliance and accuracy of information is not intrinsically (weakly) increasing or decreasing, but rather one that depends on the type of the agency. Intuitively, when an agency is soft ($\lambda$ is low) it would audit with a very low probability. For signals of a given precision $\varepsilon > 0$, agents will estimate the probability of detection and decide their income declarations accordingly. If signals became more precise (if $\varepsilon$ decreased), some agents who believed the agency was medium/tough before would now believe the agency is soft, and so would expect a lower probability of detection, which in turn makes evasion relatively more attractive and leads

\textsuperscript{19}It is important to remember that we only consider exogenous, private signals, which means –particularly– that the agency cannot manipulate the signal to its advantage. We ignore the latter possibility because taxpayers would have reasons for suspicion, and so would dismiss the agency's messages, which will be deemed "cheap talk". This is the argument of Stella (1991). For the agency's messages to have an impact, they have to be backed up by observable, costly actions (as in Stenroed et al. (2001) or Alm and McKee (2006)), which requires modelling explicitly the agency's problem of choosing said action, a problem that is not the focus of this paper.
to lower compliance. An analogous story can be used when the agency is tough (λ is high). The result is illustrated in figure 1.20

**Result 5** When signals become more precise, the tax agency audits low declarers with (weakly) higher probability, evaders get a (weakly) lower payoff, and the threshold signal Σ (weakly) decreases. Formally,

\[
\begin{align*}
(1) & \quad a^0(D^*, \lambda; \varepsilon') \leq a^0(D^*, \lambda; \varepsilon) \quad \forall \varepsilon' > \varepsilon \\
(2) & \quad E\text{u}^{\text{Evasion}}(y, s; \varepsilon') \geq E\text{u}^{\text{Evasion}}(y, s; \varepsilon) \quad \forall \varepsilon' > \varepsilon \\
(3) & \quad \Sigma(\varepsilon') \geq \Sigma(\varepsilon) \quad \forall \varepsilon' > \varepsilon
\end{align*}
\]  

**FIGURE 3 ABOUT HERE**  
**FIGURE 4 ABOUT HERE**

Intuitively, when signals become more precise (ε decreases) the tax agency audits less intensively because taxpayers believe that the agency is tough (i.e., λ > 0): indeed, \(a^0\) takes positive values only if \(\lambda > 0\), and from the previous result we know that in this case more precise signals lead to more compliance (higher \(D^*\)), which in turn means that the agency does not have to audit that intensively as before (see figure 3). The resulting lower probability of detection is behind parts 2 and 3 (figure 4 depicts the effect on the threshold \(\Sigma\)). It is worth noticing that taxpayers do not gain anything from an increase in the precision of the signals: clearly it does not affect the utility of compliant taxpayers and it decreases the utility of evaders.

**Result 6** In good years, when signals become more precise, the expected loss weakly increases if the agency is soft (λ < 0) and weakly decreases otherwise. Formally, if \(y = 1\), then

\[
\begin{align*}
\text{if } \lambda < 0, & \quad \text{then } EL(\lambda; \varepsilon') \leq EL(\lambda; \varepsilon) \quad \forall \varepsilon' > \varepsilon \\
\text{if } \lambda > 0, & \quad \text{then } EL(\lambda; \varepsilon') \geq EL(\lambda; \varepsilon) \quad \forall \varepsilon' > \varepsilon
\end{align*}
\]  

This finding is related to result 4: if signals become more precise and the agency is soft, then (from result 4) evasion increases (\(D^*\) decreases) and (from result 5) the probability of an audit \(a^0\) remains constant and equal to zero, so the number of negligence errors \((1 - D^*)(1 - a^0)\) increases and so does the expected loss.21 Analogously, if signals become more precise but the agency is medium or tough, then compliance increases (\(D^*\) increases) and the probability of an audit \(a^0\) increases, so the number of negligence errors \((1 - D^*)(1 - a^0)\) decreases and so does the expected loss.22 Figure 5 illustrate this effect.

---

20 In figures 1, 3, 4, 5 and 6 the solid/dashed lines correspond to the cases in which signals are relatively less/more precise (ε is relatively high/low).
21 This case corresponds to the second line of table 4.
22 This case corresponds to the third line of table 4.
Result 7  In good years, when signals become more precise, the expected social welfare weakly increases. Formally, if \( y = 1 \), then

\[
EW(\varepsilon') \leq EW(\varepsilon) \quad \forall \varepsilon' > \varepsilon \tag{43}
\]

Intuitively, this means that more noise is bad for society, because it leads to evasion and (most importantly) to more audits, which in this model (as in most of the literature) are just a waste of resources (unlike taxes and fines, which are transfers that do not increase or decrease total wealth). Thus, as signals become noisier, the range of \( \lambda \) for which people evade is expanded, more audits are undertaken and social welfare decreases. This is illustrated in figure 6.

These results can be rationalized in an alternative way: a lower \( \varepsilon \) can be interpreted as a higher degree of aggregation of information (or information-sharing). That is, if taxpayers shared their signals (or, in yet another alternative, if every taxpayer received \( n > 1 \) private signals), the effect would be equivalent to an increase in their precision, since the group’s average signal is expected to be closer to the true value of \( \lambda \) than the individual ones (recall that signals are on average correct –see equation 15). Thus, information sharing unequivocally leads to less audits and lower expected utility for evaders, but its effect on compliance and on the agency’s objective function depends on the type of agency. It is worth mentioning (from results 5 and 6) that an increase in the precision of signals has an asymmetric impact on the payoffs of the players: while it can benefit or harm the tax agency, it never benefits taxpayers.

3.7 Comparison with cut-off rule

In order to compare our equilibrium auditing strategy to the cut-off rule, consider the following strategy: \( a_{COR}^0 = \min \{ b(\lambda), 1 \} \) and \( a_{COR}^1 = 0 \), where the subindex \( COR \) stands for “cut-off rule”. The rationale behind \( a_{COR}^1 = 0 \) is straightforward, and \( a_{COR}^0 \) simply states that the agency audits low-income returns with a probability that is weakly increasing in its type. The key difference with our auditing strategy (that we will label “relative auditing strategy”, \( RAS \)) is that ours depends on the declarations of every taxpayer (hence the “relative” part of the name), while the \( COR \) does not. As a consequence, taxpayers play a coordination game in our model, but they do not when the cut-off rule is used.

Alternatives rules are possible, though less appealing: \( a_{COR}^1 > 0 \) is never efficient and \( a_{COR}^0 = 0 \) would lead to full evasion in good years. The functional form of \( a_{COR}^0 \) has the extra advantage of being very similar to that of \( \sigma^0* \) in proposition 2 (and lemma 3), which simplifies the comparison between rules.
If the COR is used and taxpayer $i$ declares low income, her belief about the probability of detection conditional on her signal $s_i$ is

$$E(a_{COR}^0|s_i) = \begin{cases} 
0 & \text{if } s_i < 0 \\
 s_i & \text{if } 0 < s_i < 1 \\
1 & \text{if } 1 < s_i 
\end{cases} \quad (44)$$

and so the signal that makes the taxpayer indifferent between evading and complying is $\Sigma_{COR} = P$. The probability of evasion of a high-income person is thus

$$\Pr(s_i < P|\lambda) = \begin{cases} 
1 & \text{if } \lambda < P - \varepsilon \\
\frac{\lambda - \varepsilon}{P - \varepsilon} & \text{if } P - \varepsilon < \lambda < P + \varepsilon \\
0 & \text{if } P + \varepsilon < \lambda 
\end{cases} \quad (45)$$

The expected loss of the agency is therefore

$$EL_{COR} = (1 - \gamma) (1 - \mu) a_{COR}^0 + \gamma \mu (1 - a_{COR}^0) \Pr(s_i < P|\lambda) \quad (46)$$

Focusing on the zeal errors, we obtain

$$EZ_{COR} - EZ_{RAS} = \begin{cases} 
0 & \text{if } \lambda < 0 \\
(1 - \mu) \lambda & \text{if } 0 < \lambda < 1 \\
1 - \mu & \text{if } 1 < \lambda 
\end{cases} \quad (47)$$

and so zeal errors are (weakly) higher with the cut-off rule than with the relative one. For negligence errors we get

$$EN_{COR} - EN_{RAS} = \begin{cases} 
0 & \text{if } \lambda < \Sigma - \varepsilon \\
 \mu (1 - P + \frac{1}{2\varepsilon} \lambda) & \text{if } \Sigma - \varepsilon < \lambda < \Lambda \\
 \mu (1 - \lambda) & \text{if } \Lambda < \lambda < P - \varepsilon \\
 \mu (1 - \lambda) \frac{P + \varepsilon - \lambda}{2\varepsilon} & \text{if } P - \varepsilon < \lambda < P + \varepsilon \\
0 & \text{if } P + \varepsilon < \lambda 
\end{cases} \quad (48)$$

and so negligence errors are also (weakly) higher with the cut-off rule than with the relative one. These results are summarized in the following proposition:

---

24 It is assumed throughout this section that $\varepsilon < 1 - P$. The qualitative results hold in other cases as well, but because of their number and due to space limitations we restrict our attention to just this one case. Other cases are available on request.

25 We consider the case in which $\Lambda < P - \varepsilon$. The opposite case yields the same qualitative results.
Proposition 7 The relative auditing strategy (RAS) is weakly superior to the cut-off rule (COR): the agency makes (weakly) less (negligence and zeal) errors with the RAS than with the COR.

4 Discussion

As every model, the one presented here is built upon some simplifying assumptions that make it more tractable and elegant, but also more restrictive and unrealistic.

Indeed, it is unlikely to be found in the real world the dichotomous character of income assumed here. When more than two levels of income are allowed, the probability of detection of a given individual depends on the relative position of the taxpayer’s declaration compared to the rest of the population’s: if it is among the highest ones, then the taxpayer’s probability of detection is still (weakly) increasing in the agency’s type and, under mild assumptions, (weakly) decreasing in the amount declared; if it is not, the agency knows the taxpayer is lying and audits her with the highest possible probability. When only two levels of income are considered, this policy collapses to the one presented earlier in this paper.26

Along similar lines, it is clear that the assumption of perfect correlation among taxpayers’ incomes is an implausible one. However, it is just intended to capture the fact that usually taxpayers that belong to the same category are homogeneous in most aspects, including income. Relaxing it will not change the (qualitative) results, as long as the common shocks are maintained as the main source of income variability. This ensures that there is still a significant degree of correlation among incomes and, therefore, that other taxpayers’ declarations convey useful information about the common shock that affects the category.27 Also important for the analysis is the fact that incomes within a class are more homogeneous than the signals received by its members, such that the differences among them are mainly due to disparate perceptions of the tax agency’s type. Thus, the assumption of perfect uniformity allows us to observe the effect of the fundamental uncertainty unadulterated by the presence of income heterogeneity, and so the analysis is greatly simplified.

26Also, irrespective of the levels of income allowed, if they are bounded above (i.e., $y_i \leq y_{\text{max}} \forall i \in I$), the agency would never audit those who declare $y_{\text{max}}$. In the case of unbounded domain, the probability of detection decreases as the declaration increases, as is standard in the literature.

A related matter is the one related to the observability of the common shock. It may be argued that the agency can observe many of them, probably with some delay. This is reasonable in many cases and it is not a problem for our model, as we only need taxpayers to be relatively better informed than the agency about the shock. E.g., the agency may know a given shock was positive, but not its exact intensity (low or high), while taxpayers know both sign and intensity. Normalizing the “low shock” to 0 and the “high shock” to 1, we obtain the model developed in the previous sections. An analogous normalization can be used in every other case.

27Adding an idiosyncratic shock makes our model more robust, as it eliminates the possibility of sequential auditing in which the agency audits just one person and –by the perfect correlation assumption– learns true income with certainty, and then reacts accordingly. Even comparatively small idiosyncratic shocks would impede this policy, because the agency cannot audit enough people as to find out with certainty taxpayers’ true income: the IRS has never audited more than 5% of tax returns and in recent years the audit rates have remained in the range of 1% (Bloomquist (2010)).
The importance of the partitioning of the taxpayer population into fairly homogeneous categories is highlighted by the fact that the above mentioned “relatively high correlation” condition is achieved when the category consists of agents that are very similar to each other in terms of their “observables” (age, profession, gender, etc.), since in this case their idiosyncratic shocks will be relatively small compared to the category-wide ones.\textsuperscript{28} The partitioning problem is not part of the scope of this paper, but we can identify the type of classes that favor the present model: its predictions are more likely to fit the data from classes with a large number of rather homogeneous people (e.g., unskilled manufacture workers or non-executive public servants) than the ones from small and/or heterogeneous classes.

Finally, as mentioned in the Introduction section, this analysis can be easily adapted to other applications of relevance, like the allocation of welfare benefits or the regulation of industries.

5 Conclusions

Tax evasion, the allocation of welfare benefits and the regulation of industries, among others, share a common feature: all of them are inspection games in which the tax or welfare or regulatory agency (the principal) uses the messages sent by taxpayers or benefit claimants or regulated firms (the agents) when deciding which ones of them to inspect. In order to make an informed decision, principals often bundle agents in rather homogeneous categories, without noticing that by doing this they minimize idiosyncratic shocks but simultaneously increase the relative importance of common shocks that affect all agents in the category. When we add these common shocks to the equation, we find that the principal’s optimal inspection strategy consists in auditing agents with a probability that depends not only on the agent’s own message, but also on every other agent’s. Intuitively, other agents’ messages provide contemporaneous information about the likelihood of a given shock and therefore improve the targeting proficiency of the principal.

Implementing this policy does not require new information to be gathered by the principal, just using the available information better. Yet, it changes the nature of the problem: on top of the standard cat-and-mouse game each agent plays against the principal, they also play a coordination game against each other, a game in which a negative externality between them is created by the rational, optimizing principal, a game agents would not play if the inspection strategy were not contingent on the vector of messages.

The heterogeneity in private signals eliminates the policy design difficulties that the multiplicity of equilibria seems to generate and paves the way for modelling the problem

\textsuperscript{28}These “observables” refer to variables that are exogenous to (or costly to manipulate by) the agents, and so do not include taxpayers’ current declarations.
as a **global game** which not only is more realistic, but also predicts a unique equilibrium which is consistent with empirical evidence.

Results can be extrapolated to many relevant economic problems, like the allocation of import/export quotas, the awarding of bonuses based on peer-evaluations, and the granting of research funds as a function of submitted projects, among others.

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### A Derivation of the Expected Loss Function (ELF)

From equations 9, 10 and 11, taxpayer $i$’s expected loss is given by

$$
L (a_i, d_i, y_i) = \mu (1 - a_i) (1 - d_i) y_i + (1 - \mu) a_i [1 - (1 - d_i) y_i]
$$

(49)

Since taxpayers incomes are perfectly correlated, then it becomes $L (a_i, d_i, 0) = \mu (1 - a_i)$ if $y = 0$ and $L (a_i, d_i, 1) = \mu (1 - a_i) (1 - d_i) + (1 - \mu) a_i d_i$ if $y = 1$.

Notice that (given that taxpayers are *ex-ante* identical from the agency’s perspective) those with identical declarations face the same probability of being audited. Formally, $\forall i, j \in I$, if $d_i = d_j$, then $a_i = a_j$. Since declarations can take only two values, then the agency’s choice variables are reduced to two: the probability of auditing someone who declares low income, $a^0$; and the probability of auditing someone who declares high income, $a^1$. And since taxpayers’ declaration only depend on their types (equation 19), taxpayers with the same type generate the same level of expected loss for the tax agency. Therefore, only three scenarios are possible:

- When income is low, every taxpayer declares low income and is audited with probability $a^0$: $L (a^0, 0, 0) = (1 - \mu) a^0$,
• When income is high, a fraction $1 - D^*$ of the taxpayers evade and are audited with probability $a^0$: $L(a^0, 0, 1) = \mu (1 - a^0);

• When income is high, a fraction $D^*$ of the taxpayers comply and are audited with probability $a^1$: $L(a^1, 1, 1) = (1 - \mu) a^1$.

Thus, using equation 22, the expected loss function (ELF) is simply

$$ELF = [1 - \pi(D^*, \lambda)] L(a^0, 0, 0) + \pi(D^* \lambda) [D^* L(a^1, 1, 1) + (1 - D^*) L(a^0, 0, 1)]
\quad = [1 - \pi(D^*, \lambda)] (1 - \mu) a^0 + \pi(D^* \lambda) [D^* (1 - \mu) a^1 + (1 - D^*) \mu (1 - a^0)]$$

(B) Proofs

Proof. Proposition 1: From equation 4, if $y = 0$, the utility of declaring 0 is $u(0, a_i, 0) = 0$ and the utility of declaring 1 is $u(1, a_i, 0) = -t$. Thus $u(0, a_i, 0) > u(1, a_i, 0)$ and declaring low income is a strictly dominant strategy in bad years.

Result 1: It is straightforward to note that $D^* = 0$ if $y = 0$, independently of the value $\lambda$ takes. This is reasonable and in consonance with proposition 1.

If $y = 1$, then taxpayer $i$’s optimal strategy (from equation 19) consists of evading if her private signal is low ($s_i < \Sigma$) and complying if it is high ($\Sigma < s_i$). From equation 15 and Bayes law, signals are uniformly distributed around the agency’s type $\lambda$ with posterior distribution given by equation 18: thus, the lowest possible signal is $s - \epsilon$ and the highest possible one is $s := \lambda - \epsilon$. Therefore, three cases are possible:

1. if $s \leq \Sigma$ (i.e., if $\lambda < \Sigma - \epsilon$), then every taxpayer gets a signal lower than the threshold $\Sigma$ and declares low income. Thus, in this case $D^* = 0$;

2. if $s \geq \Sigma < s$ (i.e., if $\Sigma - \epsilon < \lambda < \Sigma + \epsilon$), then some taxpayers get signals lower than the threshold $\Sigma$ and declare low income and others get signals greater than the threshold $\Sigma$ and declares high income. Thus, $D^* = \int_{\Sigma}^{\lambda+\epsilon} \frac{1}{2\pi} d\lambda = \frac{\lambda+\epsilon-\Sigma}{2\epsilon}$. Intuitively, the average declaration is equal to the fraction of taxpayers who declare high income which, in turn –and thanks to equation 19– is simply the fraction of taxpayers who receive private signals higher than the threshold $\Sigma$.

3. if $\Sigma \leq s$ (i.e., if $\Sigma + \epsilon < \lambda$), then every taxpayer gets a signal greater than the threshold $\Sigma$ and declares high income. Thus, in this case $D^* = 1$.

Lemma 1: By Bayes’ law, $\pi(D^*, \lambda) := \Pr(y = 1 | D^*, \lambda) := \frac{\Pr(y = 1, D^* | \lambda)}{\Pr(D^* | \lambda)}$. 

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From the proof of result 1 above, a not very soft tax agency (one with type \( \lambda > \Sigma - \varepsilon \)) would observe \( D^* = 0 \) when \( y = 0 \) and \( D^* > 0 \) when \( y = 1 \). Thus, \( \pi(D^* = 0, \lambda > \Sigma - \varepsilon) = \Pr(y = 1 | D^* = 0, \lambda > \Sigma - \varepsilon) = 0 \) and \( \pi(D^* > 0, \lambda > \Sigma - \varepsilon) = \Pr(y = 1 | D^* > 0, \lambda > \Sigma - \varepsilon) = 1 \). Analogously, a very soft tax agency (one with type \( \lambda < \Sigma - \varepsilon \)) would always observe \( D^* = 0 \) (both in good and in bad years). Thus, \( \Pr(D^* = 0 | \lambda < \Sigma - \varepsilon) = 1 \) and \( \Pr(y = 1, D^* = 0 | \lambda < \Sigma - \varepsilon) = \gamma \) (the unconditional probability of a good year), so that \( \pi(D^* = 0, \lambda < \Sigma - \varepsilon) = \Pr(y = 1 | D^* = 0, \lambda > \Sigma - \varepsilon) = \gamma \).

Lemma 2: From equation 20, \( \Sigma - \varepsilon < 0 \), and so if \( \lambda < \Sigma - \varepsilon \) then \( \lambda < 0 \). Combining the latter result with assumption 2, we obtain that \( b(\lambda) = 0 \) if \( \lambda < \Sigma - \varepsilon \). Thus, \( \min \{b(\lambda) ; 1\} = \min \{0 ; 1\} = 0 \).

Lemma 3: If \( D^* \in (0,1] \) then, from lemma 1 and result 1, \( D^* = \min \{\frac{\lambda + \varepsilon - \Sigma}{2 \varepsilon} ; 1\} \) and \( 1 - D^* = \max \{\frac{\Sigma + \varepsilon - \lambda}{2 \varepsilon} ; 0\} \).

Line 1: If \( \lambda \leq 0 \) then (by equation 20) \( \frac{\Sigma + \varepsilon - \lambda}{2 \varepsilon} > 0 \) (so that \( 1 - D^* > 0 \)) and \( b(\lambda) = 0 \) (from assumption 2). Hence, \( \min \{\frac{b(\lambda)}{1 - D^*} ; 1\} = 0 \) if \( \lambda \leq 0 \).

Lines 2 and 3: If \( \lambda > 0 \) then \( b(\lambda) = \lambda \) (from assumption 2) and \( 1 - D^* = \max \{\frac{\Sigma + \varepsilon - \lambda}{2 \varepsilon} ; 0\} \). Thus \( \frac{b(\lambda)}{1 - D^*} = \min \left\{ \frac{\lambda}{\Sigma + \varepsilon - \lambda - 1} ; \lim_{\lambda \to 0} \frac{\lambda}{1 - D^*} \right\} = \frac{2 \lambda}{\Sigma + \varepsilon - \lambda} \). This expression is a continuous, increasing function of \( \lambda \) for all \( \lambda \in [0, \Sigma + \varepsilon] \), it is equal to 0 when \( \lambda = 0 \), it is equal to 1 when \( \lambda = \Lambda \), and \( \lim_{\lambda \to \Sigma + \varepsilon} \frac{2 \lambda}{\Sigma + \varepsilon - \lambda} = +\infty \). Thus, if \( \lambda \geq 0 \) then \( \min \left\{ \frac{b(\lambda)}{1 - D^*} ; 1\right\} = \begin{cases} \frac{2 \lambda}{\Sigma + \varepsilon - \lambda} & \text{if } 0 \leq \lambda \leq \Lambda \\ 1 & \text{if } \Lambda \leq \lambda \end{cases} \)

Result 2: It is immediate from equations 20 and 24.

Proposition 2: Assume that the probability of a good year \( \gamma \) and/or the tax agency’s concern for negligence errors \( \mu \) are sufficiently high. Formally, \( \gamma + \mu > 1 \).

First, third, fourth and fifth lines of the proposition: From lemma 1, since \( D^* > 0 \) then \( \pi(D^*, \lambda) = 1 \) and the ELF simplifies to \( E_T(A) (L) = \mu (1 - a^0) (1 - D^*) + (1 - \mu) a^1 D^* \). Clearly, \( E_T(A) (L) \) increases with \( a^1 \) (so it is optimal to set it equal to 0 –line 1) and decreases with \( a^0 \) (so it is optimal to set it as high as possible, namely, \( a^0 = \min \left\{ \frac{b(\lambda)}{1 - D^*} ; 1\right\} \), which, by lemma 3, corresponds to the expressions in lines 3 to 5).

Second line: Two subcases need to be considered

1. If \( \Sigma - \varepsilon < \lambda \): From lemma 1 and the proof of result 1, since \( D^* = 0 \) and \( \Sigma - \varepsilon < \lambda \), then \( \pi(0) = 0 \) and the ELF becomes \( E_T(A) (L) = (1 - \mu) a^0 \). Clearly, the expected loss increases with \( a^0 \) and so it is optimal to set it equal to 0.
2. If $\lambda < \Sigma - \varepsilon$: From lemma 1 and the proof of result 1, since $D^* = 0$ and $\lambda < \Sigma - \varepsilon$, then $\pi(0) = \gamma$ and the ELF becomes $E_{TA}(L) = (1 - \gamma) (1 - \mu) a^0 + \gamma \mu (1 - a^0)$. Deriving with respect to $a^0$ we obtain $\frac{DE_{TA}(L)}{DA^0} = 1 - \mu - \gamma$, which is negative by the assumption made above. Thus, it is optimal to set $a^0$ as high as possible, namely, $a^0 = \min \{ b(\lambda); 1 \}$. From lemma 2 this simplifies to $a^0 = 0$.

Thus, both subcases lead to the same result, namely, $a^0 = 0$ if $D^* = 0$.

**Proposition 3**: By direct inspection of equation 27.

**Result 3**: From direct inspection of equation 32.

**Proposition 4**: If taxpayer $i$ receives signal $s_i = \Sigma$, then her subjective belief about the probability of an audit is, by definition, $E(a_i|\Sigma) := \int_{-\infty}^{\infty} a_i(\lambda) dG(\lambda|\Sigma)$, where $a_i(\lambda)$ and $G(\lambda|\Sigma)$ are given by equations 27 and 16, respectively. Using result 2, it simplifies to $E(a_i|\Sigma) = \int_0^\Lambda \frac{2\lambda \Lambda}{2\varepsilon + \Lambda - 1} d\lambda + \int_{\Lambda + \varepsilon}^{\Sigma + \varepsilon} \frac{1}{2\varepsilon} d\lambda$. By changing the integration variable (from $\lambda$ to $x := \Sigma + \varepsilon - \lambda$), the first term becomes $-\frac{\Sigma + \varepsilon}{1 + 2\varepsilon} + (\Sigma + \varepsilon) \ln (1 + \frac{1}{2\varepsilon})$ and the second term equals $\frac{\Sigma + \varepsilon}{1 + 2\varepsilon}$. Thus, $E(a_i|\Sigma) = (\Sigma + \varepsilon) \ln (1 + \frac{1}{2\varepsilon})$. Linearizing the logarithmic function around 1 simplifies the expression to $E(a_i|\Sigma) = \frac{\Sigma + \varepsilon}{2\varepsilon}$. Equalizing this expression to $P$ (as suggested by equation 34) yields equation 35.

**Corollary 1**: Since $\varepsilon > 0$, it is immediate from equations 35 and 33.

**Proposition 5**: As a generalization of the method used in the proof of proposition 4 above, if taxpayer $i$ receives signal $s_i$, then her subjective belief about the probability of an audit is, by definition $E(a_i|s_i) := \int_{-\infty}^{\infty} a_i(\lambda) dG(\lambda|s_i)$. Using equations 27 and 16, the expression becomes $E(a_i|s_i) =$

\[
\begin{cases}
0 & \text{if} \quad s_i \leq -\varepsilon \\
- (s_i + \varepsilon) + (\Sigma + \varepsilon) \ln (1 + \frac{s_i + \varepsilon}{2\varepsilon + \Lambda}) & \text{if} \quad -\varepsilon \leq s_i \leq \Lambda - \varepsilon \\
\frac{s_i - \Sigma}{2\varepsilon} + (\Sigma + \varepsilon) \ln \left(1 + \frac{1}{2\varepsilon}\right) & \text{if} \quad \Lambda - \varepsilon \leq s_i \leq \Lambda \\
\frac{2\varepsilon - (1 + 2\varepsilon)(\Lambda + \varepsilon - s_i)}{2\varepsilon} + (\Sigma + \varepsilon) \ln \left(\frac{(1 + 2\varepsilon)(\Lambda + \varepsilon - s_i)}{2\varepsilon \Lambda}\right) & \text{if} \quad \varepsilon \leq s_i \leq \varepsilon + \Lambda \\
1 & \text{if} \quad \varepsilon + \Lambda \leq s_i
\end{cases}
\]

which is a continuous, strictly increasing function of the private signal $s_i$ in the $(-\varepsilon, \varepsilon + \Lambda)$ interval, that takes values between 0 and 1 in the same interval. As a consequence, there is just one value of $s_i$ such that $E(a_i|s_i)$ equals $P \in \left(\frac{1}{2}, 1\right)$: for signals below this value the taxpayer’s belief is lower than $P$ and for signals above it the belief is higher than $P$. When $s_i = \Sigma$ the expression reduces to $E(a_i|\Sigma) = \frac{\Sigma + \varepsilon}{2\varepsilon}$, as shown in the proof of proposition 4 above, and –by the same proof– we know that $E(a_i|\Sigma) = P$. Combining the results from the last two sentences, proposition 5 results.
Result 4: By direct inspection of table 3 (last column).

Result 5: Part 1 can be easily be proven by direct inspection of table 3 (especially the last column). Part 2 is straightforward by noticing that $E_{\text{Ev}}(y, s; \varepsilon) = 1 - \frac{1}{\pi} a^0(D^*, \lambda; \varepsilon)$ and using part 1. Part 3 is immediate from equation 35.

Result 6: By direct inspection of table 4.

Result 7: Let us define expected social welfare as the sum of the agency’s expected net revenue and taxpayers’ expected utility. Formally,

$$EW := ENR + EU$$

$$= \pi(D^*) [D^* t + (1 - D^*) a^0 (f - c)] + [D^* u^c + (1 - D^*) u^e]$$

where $c$ is the cost of an audit and $u^c (u^e)$ is the expected utility of a compliant taxpayer (evader). Using the information in tables 3 and 4, the expected social welfare is given by the following expression:

$$EW = \begin{cases} 1 & \text{if} & \lambda < 0 \\ 1 - c\lambda & \text{if} & 0 < \lambda < \Lambda \\ 1 - Pc + \frac{\varepsilon}{2\varepsilon} \lambda & \text{if} & \Lambda < \lambda < \sum + \varepsilon \\ 1 & \text{if} & \sum + \varepsilon < \lambda \end{cases}$$

Deriving this expression with respect to $\varepsilon$ yields the result. ■

References


Table I

Title: Taxpayer i’s optimal declaration strategy

<table>
<thead>
<tr>
<th>Taxpayer i’s optimal declaration strategy</th>
<th>$y = 0$</th>
<th>$y = 1$</th>
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<tbody>
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<td>$s_i &lt; \Sigma$</td>
<td>$d_i^c = 0$</td>
<td>$d_i^c = 0$</td>
</tr>
<tr>
<td>$\Sigma &lt; s_i$</td>
<td>$d_i^c = 0$</td>
<td>$d_i^c = 1$</td>
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Table 1: Taxpayer i’s optimal declaration strategy.

Caption: Taxpayer i’s optimal declaration strategy
Table II

Title: Tax agency’s optimal auditing strategy

<table>
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<th>Tax Agency’s optimal auditing strategy</th>
<th>$D^* = 0$</th>
<th>$D^* &gt; 0$</th>
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<td>$a^0 = 0$</td>
<td>$a^0 = 0$</td>
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<tr>
<td></td>
<td>$a^1 = 0$</td>
<td>$a^1 = 0$</td>
</tr>
<tr>
<td>$0 &lt; \lambda &lt; \Lambda$</td>
<td>$a^0 = 0$</td>
<td>$a^0 = \frac{2\lambda}{\Sigma T - \lambda}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$a^1 = 0$</td>
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<td>$a^0 = 1$</td>
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<td></td>
<td></td>
<td>$a^1 = 0$</td>
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Table 2: Tax agency’s optimal auditing strategy.

Caption: Tax agency’s optimal auditing strategy
Table III

Title: Equilibrium strategies

<table>
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<th>Equilibrium strategies</th>
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<tbody>
<tr>
<td>$\lambda &lt; \Sigma - \varepsilon$</td>
<td>$d_i = 0$ \quad ($\therefore D = 0$) \quad $a_0^0 = 0$</td>
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</tr>
<tr>
<td>$\Sigma - \varepsilon &lt; \lambda &lt; 0$</td>
<td>$d_i = 0$ \quad ($\therefore D = 0$) \quad $a_0^0 = 0$</td>
<td>$d_i = \begin{cases} 0 &amp; \text{if } s_i &lt; \Sigma \ 1 &amp; \text{if } s_i &gt; \Sigma \end{cases}$ \quad ($\therefore D = \frac{\lambda + 2e(1-P)}{2e} \in (0,1)$) \quad $a_0^0 = 0; \ a_1^0 = 0$</td>
</tr>
<tr>
<td>$0 &lt; \lambda &lt; \Lambda$</td>
<td>$d_i = 0$ \quad ($\therefore D = 0$) \quad $a_0^0 = 0$</td>
<td>$d_i = \begin{cases} 0 &amp; \text{if } s_i &lt; \Sigma \ 1 &amp; \text{if } s_i &gt; \Sigma \end{cases}$ \quad ($\therefore D = \frac{\lambda + 2e(1-P)}{2e} \in (0,1)$) \quad $a_0^0 = \frac{2e\lambda}{2eP - \lambda}; \ a_1^0 = 0$</td>
</tr>
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<td>$\Lambda &lt; \lambda &lt; \Sigma + \varepsilon$</td>
<td>$d_i = 0$ \quad ($\therefore D = 0$) \quad $a_0^0 = 0$</td>
<td>$d_i = \begin{cases} 0 &amp; \text{if } s_i &lt; \Sigma \ 1 &amp; \text{if } s_i &gt; \Sigma \end{cases}$ \quad ($\therefore D = \frac{\lambda + 2e(1-P)}{2e} \in (0,1)$) \quad $a_0^0 = 1; \ a_1^0 = 0$</td>
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<td>$\Sigma + \varepsilon &lt; \lambda$</td>
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<td>$d_i = 1$ \quad ($\therefore D = 1$) \quad $a_1^0 = 0$</td>
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Table 3: Equilibrium strategies

Caption: Equilibrium strategies
Table IV

Title: Equilibrium payoffs

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<th>Equilibrium payoffs</th>
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<td>$EL = \gamma \mu$</td>
</tr>
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<td>$\Sigma - \varepsilon &lt; \lambda &lt; 0$</td>
<td>$u_i = 0$</td>
<td>$EL = 0$</td>
</tr>
<tr>
<td>$0 &lt; \lambda &lt; \Lambda$</td>
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<td>$EL = 0$</td>
</tr>
<tr>
<td>$\Lambda &lt; \lambda &lt; \Sigma + \varepsilon$</td>
<td>$u_i = 0$</td>
<td>$EL = 0$</td>
</tr>
<tr>
<td>$\Sigma + \varepsilon &lt; \lambda$</td>
<td>$u_i = 0$</td>
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Table 4: Equilibrium payoffs

Caption: Equilibrium payoffs